Holomorphy and dynamical scales in supersymmetric gauge theories

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Abstract

New recent results in supersymmetric gauge theories based on holomorphy and symmetry considerations are extended to the case where the gauge coupling constant is given by the real part of a chiral superfield. We assume here that its dynamics can be described by an effective quantum field theory. Then its vacuum expectation value is a function of the other coupling constants, viewed as chiral background superfields. This functional dependence can be determined exactly and satisfies highly non-trivial consistency checks.

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1 Introduction

In the last year, a lot of progress has been done in understanding the dynamics of $N = 1$ supersymmetric gauge field theories in four dimensions [1]. Using the principle of holomorphy introduced in [2] (see also [3], [4], [5]), new results were obtained for supersymmetric QCD in case where the number of flavours N_f is larger or equal to the number of colours [6],[7]. New exact superpotentials were derived with a highly non-trivial dynamics [8] and with a possible impact on supersymmetry breaking [9].

Some of the results were checked by dynamical instanton computations [10]. The methods were further generalized in order to accommodate for soft-breaking terms [11], [12].

The two main tools used can be described as follows.
(1) Holomorphy: The Wilsonian superpotential W_{eff} is a holomorphic function of the fields and coupling constants. This is equivalent to consider the coupling constants as chiral background fields.
(2) Symmetries: W_{eff} is invariant under all the symmetries of the model. If some of the symmetries are explicitly broken by the coupling constants, we assign transformation laws to the coupling constants such as to restore the full symmetry group. The anomalous symmetries can be treated on the same footing by assigning specific transformation laws to the scale(s) Λ of the gauge group(s) [8].

In case where supersymmetry is not spontaneously broken, the combination of these two principles implies that the vacuum expectation values of chiral fields in W_{eff} are holomorphic functions of the coupling constants compatible with the symmetries. This is a powerfull result which will be largely used in the following.

The purpose of this letter is to apply these methods to the case where the gauge coupling constant is promoted to a dynamical chiral superfield (the dilaton superfield of effective superstring theories). If the dynamics of this additional field can be described by an effective quantum field theory, then by use of the above-mentioned principles, its vacuum expectation value is a function of the other coupling constants of the theory. This is a strong assumption; usually it is assumed that the gauge coupling is determined by

the high energy physics. The new hypothesis gives new relations, and their consistency with the already known dynamic relations is a highly non-trivial necessary consistency check. Still, our considerations can only prove in which case this assumption is wrong; it can well be that all our consistency checks are verified and for other reasons, the gauge coupling is fixed by the high energy dynamics.

Section 2 describes the consequences of this assumption in the case of supersymmetric QCD with gauge group $\mathrm{SU}(N_c)$ and number of flavours $N_f \geq N_c$ (with or without an additional field in the adjoint of the gauge group) and gauge group $SO(N_c)$. Some consistency checks are worked out in detail, related to the decoupling of massive flavors or of the adjoint field, the NSVZ beta function to all orders [13] and the non-abelian duality of ref. [7].

In section 3 an effective Lagrangian analysis is performed, which supports and clarifies the results of section 2.

Finally some conclusions are drawn together with some comments.

2 Dynamical gauge coupling constant and holomorphy

The model to be considered below is a simple generalization of supersymmetric QCD with N_f flavours of quarks, Q^i in the N_c representation of $\mathrm{SU}(N_c)$ gauge group and $\widetilde{Q}_{\bar{\imath}}$ in the $\overline{N}_c$ representation. For simplicity reasons, we begin with the case $N_F = N_c = N$. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\mathrm{Kin}} + \mathcal{L}_{\mathrm{couplings}} \ ,$$

where

$$\begin{aligned} \mathcal{L}_{\mathrm{Kin}} &= \int d^4\theta \left[Q^+ \mathrm{e}^V Q + \widetilde{Q}\, \mathrm{e}^{-V} \widetilde{Q}^+ + K\left(S^+, S\right)\right] + \\ &\quad + \left(\int d^2\theta \frac{1}{4} S W^\alpha W_\alpha + h.c. \right) \end{aligned} \tag{1}$$

and

$$\mathcal{L}_{\mathrm{couplings}} = \int d^2\theta \left(m_i^{\bar{\imath}} Q^i \widetilde{Q}_{\bar{\imath}} + bB + \widetilde{b}\widetilde{B} \right) + h.c. \ . \tag{2}$$

The Wilson gauge coupling constant is $\frac{1}{g_W^2} = \langle ReS \rangle$, where S is a chiral superfield. In (2), we introduced the composite chiral superfields (the "baryons")

$$B = \frac{1}{N!}\epsilon_{i_1 \ldots i_N}\, Q^{i_1} \ldots Q^{i_N}, \quad \widetilde{B} = \frac{1}{N!}\epsilon^{\bar{\imath}_1 \ldots \bar{\imath}_N}\, \widetilde{Q}_{\bar{\imath}_1} \ldots \widetilde{Q}_{\bar{\imath}_N} \;, \tag{3}$$

$b, \widetilde{b}$ are associated sources and $m_i^{\bar{\imath}}$ is a quark mass matrix in the flavour space $(i, \bar{\imath} = 1, \ldots, N_f)$.

The global symmetry of the model, in the sense described in the Introduction, is

$$G = \mathrm{SU}(N_f) \times \mathrm{SU}(N_f) \times U(1)_B \times U(1)_R \times U'(1)_R \tag{4}$$

where $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$ is the flavour chiral symmetry and $U(1)_B$ the baryon conservation. The most important point in our discussion are the two R-symmetries $U(1)_R \times U'(1)_R$. The $U(1)_R$ symmetry is always spontaneously broken and acts on the fields and couplings as follows:

$$\begin{aligned}
\left(Q, \widetilde{Q}\right)\left(\theta' = \mathrm{e}^{-\frac{3i\beta}{2}}\theta\right) &= \mathrm{e}^{-i\beta}\left(Q, \widetilde{Q}\right)(\theta)\,, \\
W'_\alpha(\theta') &= \mathrm{e}^{\frac{-3i\beta}{2}} W_\alpha(\theta)\,,\ S'(\theta') = S(\theta) + \frac{ibo}{8\pi^2}\beta, \\
(m')_i^{\bar{\imath}} &= \mathrm{e}^{-i\beta} m_i^{\bar{\imath}},\ \left(b, \widetilde{b}\right)' = \mathrm{e}^{i(N-3)\beta}\left(b, \widetilde{b}\right).
\end{aligned} \tag{5}$$

The non-linear transformation of the field S, with coefficient $b_0 = 3N_c - N_f = 2N$ in our case, cancels the one-loop triangle anomalies and restores the symmetry.

The $U'(1)_R$ symmetry is the anomaly-free combination of the usual axial symmetry and an R-symmetry. Its action is

$$\begin{aligned}
\left(Q, \widetilde{Q}\right)\left(\theta' = \mathrm{e}^{i\gamma}\theta\right) &= \left(Q, \widetilde{Q}\right)(\theta)\,, \\
W'_\alpha(\theta') &= \mathrm{e}^{i\gamma} W_\alpha(\theta)\,,\ S'(\theta') = S(\theta)\,, \\
(m')_i^{\bar{\imath}} &= \mathrm{e}^{2i\gamma} m_i^{\bar{\imath}},\ \left(b, \widetilde{b}\right)' = \mathrm{e}^{2i\gamma}\left(b, \widetilde{b}\right).
\end{aligned} \tag{6}$$

The low-energy effective theory is described by the gauge invariant fields $B, \widetilde{B}$ introduced in (3), the meson fields $M_{\bar{\imath}}^i = Q^i \widetilde{Q}_{\bar{\imath}}$ and the gauge coupling

superfield S (the glueball superfield U $=$ tr$W^\alpha W_\alpha$ is introduced also for convenience). Using the principles (1) and (2) in the Introduction, their vacuum expectation values are given by

$$\begin{aligned}
\left\langle M^i_{\bar{\imath}}\right\rangle &= C_M \left(\frac{\det m}{b\widetilde{b}}\right)^{\frac{1}{N-2}} \left(\frac{1}{m}\right)^i_{\bar{\imath}} \ , \\
\langle B\rangle &= C_B \left(\frac{\det m}{b^{N-1}\widetilde{b}}\right)^{\frac{1}{N-2}} \ , \left\langle \widetilde{B}\right\rangle = C_{\widetilde{B}} \left(\frac{\det m}{b\widetilde{b}^{N-1}}\right)^{\frac{1}{N-2}} \ , \\
\langle U\rangle &= C_U \cdot \left(\frac{\det m}{b\widetilde{b}}\right)^{\frac{1}{N-2}} \equiv \Lambda^3_{N,0} \ , \\
\langle S\rangle &= C_S - \frac{1}{4\pi^2(N-2)} \ln \frac{\det m}{\mu^{N(N-2)}\left(b\widetilde{b}\right)^{\frac{N}{2}}} \ .
\end{aligned} \tag{7}$$

In (7), $C_M ... C_S$ are some numbers to be more constrained by dynamical considerations, $\Lambda_{N,0}$ is the scale of the theory in case where all the quarks are heavy and decouples and μ is a mass scale introduced in order to restore the dimensions in $\langle S\rangle$, consistently interpreted as a renormalization scale in the following. The dynamical scale of the theory $\Lambda_{N,N}$ is defined, as usual, by the 1-loop running of the Wilson gauge coupling constant:

$$\Lambda_{N,N} = \mu \ \mathrm{e}^{-\frac{4\pi^2\langle S\rangle}{N}} = C_\Lambda \cdot \left[\frac{\det m}{\left(b\widetilde{b}\right)^{\frac{N}{2}}}\right]^{\frac{1}{N(N-2)}} \ , \tag{8}$$

where $C_\Lambda = \mathrm{e}^{-\frac{4\pi^2 C_S}{N}}$. One remark concerning the comparison of $< M^i_{\bar{\imath}} >$, $< B >$, $< \widetilde{B} >$ in (7) with the results of ref. [6] is in order. Using the result (8) of our additional assumption in eqs. (4.12)- (4.15) of ref. [6], we find that the power series in eqs. (4.14) and (4.15) in [6] collapse to only one term and the final result is qualitatively of the form (7). We will come back to this point in the next paragraph. Combining (7) and (8) in a straightforward way we get

$$\Lambda_{N,0} = \frac{C_U^{1/3}}{C_\Lambda^{2/3}} (\det m)^{\frac{1}{3N}} \Lambda^{2/3}_{N,N} = \frac{C_U^{1/3}}{C_\Lambda^{N/3}} \left(b\widetilde{b}\right)^{1/6} \Lambda^{\frac{N}{3}}_{N,N} \ . \tag{9}$$

The eqs. (9) express the well-known relations between $\Lambda_{N,0}$ and $\Lambda_{N,N}$ in two particular cases. Namely, if $b, \widetilde{b} \to 0$ a direct one-loop computation gives $\Lambda_{N,0} = (\det m)^{\frac{1}{3N}} \Lambda_{N,N}^{\frac{2}{3}}$. If $m \to 0$, a similar computation gives $\Lambda_{N,0} = \left(b\widetilde{b}\right)^{\frac{1}{6}} \Lambda_{N,N}$ [6]. Our expression (8) actually encodes both limits in an exact formula. The correct step function decoupling in (9) fixes the coefficients $C_U = C_\Lambda = 1$.

An additional dynamical constraint comes from the Konishi anomaly (see [4]), which in our case reads

$$\left\langle m_i^{\bar{\imath}} M_{\bar{\imath}}^i + bB \right\rangle = - \langle U \rangle \ , \langle bB \rangle = \left\langle \widetilde{b}\widetilde{B} \right\rangle \ . \tag{10}$$

This results in $C_B = C_{\widetilde{B}}$ and $C_M + C_B = -1$. To summarize, consistency of (7) with the decoupling theorem and the Konishi anomaly (10) fix four of the five unknown coefficients in (7),

$$C_U = C_\Lambda = 1 \ , C_M + C_B = -1 \ , C_B = C_{\widetilde{B}} \ . \tag{11}$$

It is interesting to note that using (8) and (11) into (7) we can write

$$\begin{aligned} \left\langle M_{\bar{\imath}}^i \right\rangle &= C_M \Lambda_{N,N}^2 (\det m)^{\frac{1}{N}} \left(\frac{1}{m}\right)_{\bar{\imath}}^i \ , \qquad (12) \\ \langle B \rangle &= C_B \Lambda_{N,N}^N \left(\frac{\widetilde{b}}{b}\right)^{1/2} \ , \left\langle \widetilde{B} \right\rangle = C_B \Lambda_{N,N}^N \left(\frac{b}{\widetilde{b}}\right)^{1/2} \ . \end{aligned}$$

These relations were usually obtained in some limits, the first in the $b, \widetilde{b} \to 0$ limit and the second line in the $m \to 0$ limit [6]. Hence, considering the gauge coupling as a dynamical field seems to somehow relate the two limits where $m \to 0$ and $b, \widetilde{b} \to 0$.

We turn now to the case $N_f > N_c$. The superpotential in this case reads

$$W_{\text{couplings}} = m_i^{\bar{\imath}} Q^i \widetilde{Q}_{\bar{\imath}} + \sum_a m_a Q^{N_c+a} \widetilde{Q}_{N_c+a} + bB + \widetilde{b}\widetilde{B} \ , \tag{13}$$

where $i, \bar{\imath} = 1, ..., N_c$, $a = 1, ..., N_f - N_c$ and $\left(B, \widetilde{B}\right) = \left(B^{N_c+1,...,N_f}, \widetilde{B}_{N_c+1,...,N_f}\right)$. We therefore added specific source terms in (13) for simplicity reasons. The global symmetry group in this case is

$$
\begin{aligned}
G &= SU\left(N_f\right)\times SU\left(N_f\right)\times U\left(1\right)_B\times\left(\prod_a U\left(1\right)_{Ba}\right)\times \\
&\quad \times U\left(1\right)_R\times U'\left(1\right)_R\times\left(\prod_a U\left(1\right)_{Ra}\right).
\end{aligned} \tag{14}
$$

In (14), $U\left(1\right)_{Ba}$ and $U\left(1\right)_{Ra}$ are the baryon number conservation of the heavy quarks and R-symmetries to be displayed below.

The $U\left(1\right)_R$ symmetry acts as in (5) with the appropriate beta function coefficient $b_0 = 3N_c - N_f$. The action of $U'\left(1\right)_R$ is

$$
\begin{aligned}
\left(Q,\widetilde{Q}\right)\left(\theta'=\mathrm{e}^{i\gamma}\theta\right) &= \mathrm{e}^{\frac{i\left(N_f-N_c\right)\gamma}{N_f}}\left(Q,\widetilde{Q}\right)\left(\theta\right), \\
W'_\alpha\left(\theta'\right) &= \mathrm{e}^{i\gamma}W_\alpha\left(\theta\right)\ ,S'\left(\theta'\right)=S\left(\theta\right)\ , \\
m' &= \mathrm{e}^{\frac{2iN_c\gamma}{N_f}}m\ ,\ \left(b,\widetilde{b}\right)'=\mathrm{e}^{\frac{i\left(N_c^2-N_cN_f+2N_f\right)\gamma}{N_f}}\left(b,\widetilde{b}\right)\ ,
\end{aligned} \tag{15}
$$

where $Q,\widetilde{Q}$ (m) are the set of all quark fields (masses) of the theory. The action of $U(1)_{Ra}$ is given by

$$
\begin{aligned}
\left(Q^i,\widetilde{Q}_{\bar{\imath}}\right)\left(\theta'=\mathrm{e}^{i\delta}\theta\right) &= \mathrm{e}^{i\delta}\left(Q^i,\widetilde{Q}_{\bar{\imath}}\right)\left(\theta\right)\ , \\
\left(Q^{N_c+b},\widetilde{Q}_{N_c+b}\right)\left(\theta'\right) &= \mathrm{e}^{-i\left(N_c\delta_{ab}-1\right)\delta}\left(Q^{N_c+b},\widetilde{Q}_{N_c+b}\right)\left(\theta\right)\ , \\
W'_\alpha\left(\theta'\right) &= \mathrm{e}^{i\delta}W_\alpha\left(\theta\right)\ ,S'\left(\theta'\right)=S\left(\theta\right)\ , \\
\left(m_i^{\bar{\imath}}\right)' &= m_i^{\bar{\imath}}\ ,m'_b=\mathrm{e}^{2iN_c\delta_{ab}}m_b\ , \\
\left(b,\widetilde{b}\right)' &= \mathrm{e}^{i\left(2-N_c\right)\delta}\left(b,\widetilde{b}\right)\ .
\end{aligned} \tag{16}
$$

Using the same strategy as before, we get

$$
\left\langle S\right\rangle = -\frac{4}{N_c-2}\ln\frac{\det m_i^{\bar{\imath}}}{\mu^{\frac{\left(N_c-2\right)\left(3N_c-N_f\right)}{2}}\left(b\widetilde{b}\right)^{\frac{N_c}{2}}\prod_a m_a^{\frac{N_c-2}{2}}}\ . \tag{17}
$$

Defining the dynamical scale $\Lambda_{N_c,N_f}=\mu\cdot\mathrm{e}^{-\frac{8\pi^2\left\langle S\right\rangle}{3N_c-N_f}}$ and comparing with Λ_{N_c,N_c} in (8), we get the relation

$$\Lambda_{N_c,N_c} = \left(\prod_a m_a^{\frac{1}{2N_c}}\right) \cdot \Lambda_{N_c,N_f}^{\frac{3N_c-N_f}{2N_c}} \ . \tag{18}$$

The eq. (18) is exactly that we would get in a one-loop computation with threshold effects at the scales m_a. One can also check that in this case

$$\langle U \rangle = \left(\frac{\det m_i^{\bar{i}}}{b\tilde{b}}\right)^{\frac{1}{N_c-2}} \equiv \Lambda^3_{N_c,0} \tag{19}$$

and we get the well known equation $\Lambda_{N_c,0} = \left(\det m_i^{\bar{i}} \cdot \prod_a m_a\right)^{\frac{1}{3N_c}} \cdot \Lambda_{N_c,N_f}^{\frac{3N_c-N_f}{3N_c}}$.
It is interesting to examine eq.(17) closer in connection with the measurable gauge coupling constant g. It is known that Λ_{N_c,N_f}, g and g_W are related through [3],[13]

$$\Lambda_{N_c,N_f} = \mu \ \mathrm{e}^{-\frac{8\pi^2}{\left(3N_c-N_f\right)g_W^2}} = \frac{\mu}{g^{\frac{2N_c}{3N_c-N_f}}} \ \mathrm{e}^{-\frac{8\pi^2}{\left(3N_c-N_f\right)g^2}} \ . \tag{20}$$

Combining (17) and (20) we find the exact renormalization group evolution of g

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_X)} - \frac{(3N_c-N_f)}{8\pi^2} \ln\left[\left(\frac{g(\mu)}{g(M_X)}\right)^{\frac{2N_c}{3N_c-N_f}} \ Z_Q^{\frac{N_f}{3N_c-N_f}} \ \frac{M_X}{\mu}\right] \ . \tag{21}$$

In (21), M_X is a high scale and Z_Q is defined by $m(\mu) = Z_Q^{-1} \ m(M_X), b(\mu) = Z_Q^{-\frac{N_c}{2}} b(M_X)$, where we denote by m the set of the masses $m_i^{\bar{i}}, m_a$. Differentiating (21) with respect to μ and defining the anomalous dimension $\gamma_Q = -\frac{\partial \ln Z_Q}{\partial \ln \mu}$, we find the exact beta function

$$\beta(g) = -\frac{g^3}{16\pi^2} \ \frac{3N_c - N_f + N_f \ \gamma_Q}{1 - \frac{N_c}{8\pi^2} g^2} \ . \tag{22}$$

The expression (22) is known since a long time [13], [3] to be the beta function in all orders in supersymmetric QCD. We consider this as an independent consistency check of our relation (17).

The one-loop running of g_W is easily understood taking into account that the holomorphic parameters $m, b, \tilde{b}$ entering into eq.(17) are not physical,

being related to the physical ones by the renormalization constant Z_Q defined above. By the non-renormalization theorem, $m, b, \widetilde{b}$ do not run. Hence one recovers the usual one loop running

$$\frac{1}{g_W^2\left(\mu\right)} = \frac{1}{g_W^2\left(M_X\right)} - \frac{3N_c - N_f}{8\pi^2} \ln \frac{M_X}{\mu} \ . \tag{23}$$

Finally notice that the gaugino condensation scale $\langle U \rangle = \Lambda^3_{N_c,0}$ is renormalization group invariant, as it should be.

Notice that for $N_f < N_c$ the baryons and the sources $b, \widetilde{b}$ do not exist. In this case it is easy to check that there are more R symmetries than chiral background parameters and it is impossible to write down expressions similar to (8), (17)-(18). We therefore recover the well-known runaway behaviour in SQCD for $N_f < N_c$.

These results are easily generalized to other models. We checked, for example, the case of an additional chiral superfield X in the adjoint of the gauge group studied in [15], or models with gauge groups $SO\left(N_c\right)$ studied in [7],[16]. In the first case, the superpotential of the model is (k is fixed)

$$W_{\text{couplings}} = g_k Tr X^{k+1} + m_i^{\bar{\imath}} Q^i \widetilde{Q}_{\bar{\imath}} + \sum_a m_a Q^{N_c+a} \widetilde{Q}_{N_c+a} + bB + \widetilde{b}\widetilde{B} \ . \tag{24}$$

The holomorphy and the symmetries give

$$\left[\Lambda^{(X)}_{N_c,N_f}\right]^{(2N_c-N_f)} = \left(\det m_i^{\bar{\imath}} \prod_a m_a\right)^{-1} g_k^{-\frac{2N_c}{k+1}} \left[\frac{\det m_i^{\bar{\imath}}}{b\widetilde{b}}\right]^{\frac{2N_c}{(N_c-2)(k+1)}} \ . \tag{25}$$

The gaugino condensation scale is the same as before, eq.(19). In the second case, we have $2N_f$ quark fields Q^i. Defining the baryons B as in (3) and considering the superpotential

$$W_{\text{couplings}} = \frac{1}{2} m_{ij} Q^i Q^j + \sum_a m_a Q^{N_c+a} Q^{N_c+a} + bB \ , \tag{26}$$

we get

$$\Lambda^{(SO(N_c))}_{N_c,N_f} = \left(b^2 \prod_a m_a\right)^{-\frac{1}{3N_c-N_f-6}} \tag{27}$$

and the gaugino condensation scale is $\Lambda_{N_c,0}^{(SO(N_c))} = \left(\frac{\det m_{ij}}{b^2}\right)^{\frac{1}{3(N_c-2)}}$.

In all these cases, the expression of the dynamical scale as a function of the background parameters gives a correct decoupling of heavy fields and is compatible with the beta function for the gauge coupling to all orders [13].

In order to say more about the values of the dynamical scales, in particular the gaugino condensation scale $\Lambda_{N_c,0}$ in (19), we must know the parameters m, b, $\widetilde{b}$. I give here a string-motivated example $m \sim m_{3/2}$, where $m_{3/2}$ is the gravitino mass and $(b,\widetilde{b}) \sim \frac{1}{M_P^{N_c-3}}$, where M_P is the Planck mass. In this case, in the $N_c \to \infty$ limit, for all the models discussed above, we get from (19) $\langle\lambda\lambda\rangle = \Lambda_{N_c,0}^3 \sim m_{3/2} M_P^2$, as in the usual gaugino condensation scenario in supergravity. Therefore, there is a hope that this way of fixing dynamical scales is of phenomenological relevance.

Some remarks can be made concerning the duality proposed in [7], where it is argued that, for $\frac{3N_c}{2} < N_f < 3N_c$, the original "electric" theory with gauge group $SU(N_c)$ and N_f flavours is dual to a "magnetic" theory with gauge group $SU(N_f - N_c)$ and N_f quark flavours (similar dualities hold for $SO\,(N_c)$ or $SP\,(N_c)$ gauge groups). The two different theories give the same physics if there is a non-trivial IR fixed point, where the two theories are conformally invariant.

At this point, the parameters (and the fields) acquires anomalous dimensions and we can define $m^* = \Lambda_{N_c,N_f}^{\frac{3N_c-N_f}{N_f}}\, m\ ,\ \left(b,\widetilde{b}\right)^* = \Lambda_{N_c,N_f}^{\frac{\left(3N_c-N_f\right)N_c}{2N_f}}\left(b,\widetilde{b}\right)$. Combining these fixed-point parameters with the expression of the dynamical scale Λ_{N_c,N_f} obtained from (17), we get

$$\prod_{a=1}^{N_f-N_c} m_a^* = \left[\frac{\det m_i^{*\bar{\imath}}}{\left(b\widetilde{b}\right)^{\frac{N_c}{2}}}\right]^{\frac{2}{N_c-2}} \quad . \tag{28}$$

Notice that the scale Λ cancelled out in (28), as it should in a conformally invariant theory. Eq.(28) defines an infrared surface, obtained as a result of considering Λ_{N_c,N_f} as a dynamical field.

In the magnetic theory, the symmetries and a correct decoupling force us to introduce a new mass parameter μ_0 with charge 1 under $U(1)_R$, in order to get a correct dynamical scale $\widetilde{\Lambda}_{N_f-N_c,N_f}$. The result is

$$\widetilde{\Lambda}_{N_f-N_c,N_f} = (-1)^{\frac{N_f-N_c}{2N_f-3N_c}} \mu_0^{\frac{N_f}{2N_f-3N_c}} \left(\prod_a m_a\right)^{\frac{1}{2N_f-3N_c}} \left[\frac{\det m_i^{\bar{i}}}{\left(b\widetilde{b}\right)^{\frac{N_c}{2}}}\right]^{-\frac{2}{(N_c-2)\left(2N_f-3N_c\right)}} \tag{29}$$

and combined with (17) gives $\Lambda_{N_c,N_f}^{3N_c-N_f}\ \widetilde{\Lambda}\,_{N_f-N_c,N_f}^{2N_f-3N_c} = (-1)^{N_f-N_c}\,\mu_0^{N_f}$, in agreement with [7]. The scale μ_0, introduced for other reasons in [7], is needed here in order to satisfy the decoupling theorem.

Hence we conclude that our main assumption, the gauge coupling to be a dynamical chiral superfield, is compatible with the duality introduced in [7] and leads to the appearance of IR surfaces of type (28).

3 Effective Lagrangian approach for $N_f = N_c = N$.

The symmetries of the supersymmetric QCD for $N_f < N_c$ are sufficient in order to exactly determine the low-energy effective superpotential [14]. For $N_f \geq N_c$ some additional, dynamical information must supplement the symmetry behaviour. We will use in the following some instanton arguments to reduce the freedom. The Lagrangian (1), (2) has a global symmetry only broken by the $\left(b,\widetilde{b}\right)$ sources. It acts as $\left(\lambda, Q, \widetilde{Q}\right) \to \mathrm{e}^{i\beta}\left(\lambda, Q, \widetilde{Q}\right), \left(b,\widetilde{b}\right) \to \mathrm{e}^{(2-N_c)i\beta}\left(b,\widetilde{b}\right)$ and leaves invariant the gauge fields, the fermions $\psi_Q, \widetilde{\psi}_Q$ and the mass matrix $m_i^{\bar{i}}$.

The instanton effects then give rise in general to the formula

$$\left\langle \det M_{\bar{i}}^{i} - B\widetilde{B}\right\rangle = \Lambda_{N,N}^{2N}\sum_{n=0}^{\infty} a_n \frac{\left(b\widetilde{b}\right)^{\frac{nN}{N-2.}}\Lambda_{N,N}^{2N}}{(\det m)^{\frac{2n}{N-2}}} \equiv \Lambda^{2N}\cdot f\ , \tag{30}$$

where a_n is the coefficient of $N+1$ instantons contribution. The formula (30) is also compatible with all the symmetries (4). In the $\left(b,\widetilde{b}\right) \to 0$ limit (30) becomes the equation for the quantum moduli space ($N_f = N_c$) introduced in [6]. The coefficient a_0 was computed in the massless case to be $a_0 = 1$ [10].

Using the symmetries (4) and the formula (30), we arrive at the following effective superpotential:

$$W_{\rm eff} = m_i^{\bar{\imath}} M^i_{\bar{\imath}} + bB + \widetilde{b}\widetilde{B} + U \ln \frac{\det M^i_{\bar{\imath}} - B\widetilde{B}}{\Lambda^{2N}_{N,N} \cdot f(x)} \; , \tag{31}$$

where $x = \frac{(b\widetilde{b})^{\frac{N}{N-2}} \Lambda^{2N}_{N,N}}{(\det m)^{\frac{2}{N-2}}}$, and the field S is encoded in $\Lambda_{N,N}$ using (8). Notice that by a change of variables we can move the function $f(x)$ from the last term in (31) in the first two terms, such as to keep valid the equation of the quantum moduli space introduced in [6]. Minimizing $W_{\rm eff}$ with respect to $S, M^i_{\bar{\imath}}, B, \widetilde{B}$ and U we find the vacuum structure.

Defining by x_0 and f_0 the point and the value of the function f at the minimum, we find the equations

$$\begin{aligned}
\Lambda_{N,N} &= \left[\frac{x_0 \det m}{\left(b\widetilde{b}\right)^{\frac{N}{2}}} \right]^{\frac{1}{N(N-2)}} \quad , \; \langle U \rangle = g^{-1} \left(\frac{\det m}{b\widetilde{b}} \right)^{\frac{1}{N-2}} \quad , \\
\left\langle M^i_{\bar{\imath}} \right\rangle &= (-1)^{\frac{1}{N-1}} \left[x_0^{\frac{2}{N-2}} f_0 \cdot g \right]^{\frac{1}{N-1}} \left(\frac{1}{m} \right)^i_{\bar{\imath}} \left(\frac{\det m}{b\widetilde{b}} \right)^{\frac{1}{N-2}} \quad , \\
\langle B \rangle &= g \; x_0^{\frac{2}{N-2}} \; f_0 \cdot \left(\frac{\det m}{b^{N-1}\widetilde{b}} \right)^{\frac{1}{N-2}} \quad , \; \left\langle \widetilde{B} \right\rangle = g \; x_0^{\frac{2}{N-2}} \; f_0 \cdot \left(\frac{\det m}{b\widetilde{b}^{N-1}} \right)^{\frac{1}{N-2}} \quad .
\end{aligned} \tag{32}$$

g is a function of x_0 and f_0 defined by the equation

$$(-1)^{\frac{1}{N-1}} x_0^{\frac{2}{(N-1)(N-2)}} f_0^{\frac{1}{N-1}} g^{\frac{N}{N-1}} + x_0^{\frac{2}{N-2}} f_0 \; g^2 = -1. \tag{33}$$

The results (32), (33) automatically fulfill the Konishi anomaly (10). As we showed in the preceding paragraph, a correct decoupling of massive quark flavours ask for $x_0 = 1, g = 1$. The comparison of (32), (33) with (7), (11) allow the identification $C_B = f_0 \; , \; C_M = (-1)^{\frac{1}{N-1}} f_0^{\frac{1}{N-1}}$.

Hence, what we need in order to explain in an effective Lagrangian approach the results of the previous paragraph is the presence of non-linear terms in the sources $m, b, \widetilde{b}$. This would be an exception of the linearity principle postulated in [8]. Nevertheless, the only change in the results is in the stabilization of the dynamical scales, all other dynamics being essentially the same. Alternatively, we can redefine $\Lambda'_{N,N} = f(x)^{1/2N} \Lambda_{N,N}$ by a non-linear

change of variables, in order to recover the usual effective Lagrangian of SQCD for $N_f = N_c$ and all the known results [6], [7], [15], [16] (for fixed Λ'). Then, the runaway behaviour would be just an artifact due to the singular value of the Jacobian of the change of variables.

4 Conclusions

The main goal of this letter is to apply methods based on holomorphy and symmetries to the case where the gauge coupling constant is promoted to a chiral superfield (the dilaton superfield). If its dynamics can be described by an effective field theory, then its vacuum expectation value can be exactly determined. In this way the dynamical scale of the theory is exactly computed in $SQCD$ for $N_f \geq N_c$, $SQCD$ with an additional chiral field X in the adjoint of the gauge group $SU(N_c)$ and models with $SO(N_c)$ gauge groups. Our main results (8), (17), (25), (27) highly constrain the dynamics and satisfy non-trivial consistency checks, for example (9), (12), (18), (22) and (28). For $N_f < N_c$ we find the usual runaway behaviour.

The actual value of the gaugino condensation scale (19) is shown to be potentially of phenomenological interest in the limit $N_c \to \infty$, for hidden sector models of supersymmetry breaking in supergravity.

In an effective Lagrangian approach, these results can be recovered if there are non-linear terms in the chiral background sources (2), (13), (24) and (26). Usually, these non-linear terms are eliminated in the literature by a redefinition of the dynamical scale, which is perfectly justified in $N = 1$ supersymmetric theories with a fixed gauge coupling constant (for a discussion on this point, see [17]). We argued here that if the gauge coupling is considered as a dynamical field, we must keep the non-linear structure and the picture which emerges is consistent.

We hope the results of this letter to be relevant for the dilaton stabilization problem of the effective string theories.

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